Improved Oxygen Mass Transfer Modeling for Diffused/Subsurface Aeration Systems

The mass transfer analysis of the oxygen transfer performance of diffused air or subsurface mechanical aeration systems has progressed very little over the past 20 years. The recently-developed ASCE Standard method for determination of the oxygen mass transfer performance of diffused or subsurface aeration systems is based on a greatly oversimplified mass transfer model. Although the *ASCE Standard* can be used to empirically evaluate point performance conditions, it does not provide a meaningful representation of the actual mass transfer process and is not capable of accurately assessing or predicting performance under changing operating or environmental conditions. A new oxygen mass transfer model has been developed which is a fundamentally more rigorous description of the actual mass transfer process in diffused aeration systems. This model can be confidently used to predict aerator performance under changing operating and environmental conditions. The model is easily adapted to numerical computer solution for routine aeration system performance evaluation as well as process design.

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Introduction

Much of the effort in the field of oxygen transfer and aeration systems for secondary wastewater treatment over the past decade has centered around the development of an "oxygen transfer standard" for the measurement and evaluation of the oxygen transfer efficiency of aeration devices in clean tap water. The *American Society of Civil Engineers (ASCE) Standard* (1984) was developed to organize and systematize the experimental technique and calculation procedures for determining the oxygen transfer efficiency of all types of aeration systems, including diffused air and mechanical subsurface aeration systems as well as surface aerators. The primary motivation for the development of this "standard" was to enable meaningful and accurate measurement of the relative performance capabilities of these various types of aeration systems. This was accomplished through the use of precisely-defined testing techniques and conditions, and fully-prescribed methods for the analysis and interpretation of the experimental data from these tests. The use of the ASCE Standard would then enable meaningful, relative comparison of the oxygen transfer efficiency of different types of aeration systems using a commonly-accepted set of "standard" testing methods and performance criteria.

It would appear that much has been accomplished through the development of the ASCE Standard. Certainly the testing and reporting of aeration system performance in clean tap water has been brought from a rather chaotic status in the mid-1970's to a reasonable level of standardization and meaningful performance testing at present. The main drawback in the current ASCE Standard, however, is the use of an oversimplified and unrealistic oxygen mass transfer model which is used to describe all manner and types of aeration systems. This oversimplified model results in essentially a completely-empirical characterization of aerator performance which has meaning only under the exact set of operating conditions and aerator performance parameters tested. The "standard" method performance data cannot be used for predicting or assessing the impact of changes in process design parameters or operating conditions on aerator performance and must also rely on a semiempirical set of factors to extrapolate performance from the unsteady-state clean tap water test results to actual performance under steady-state process design operating conditions. The strictly empirical nature of the "standard" method oxygen mass transfer model also forces excessive reliance on a purely statistical analysis and interpretation of the experimental test data, which further complicates the actual test procedure and data analysis. This aspect of the "standard" method also causes a great deal of confusion on the part of users.

In short, it is felt that the ASCE Standard method could be substantially improved by the use of a more fundamentally-rigorous oxygen mass transfer model. All of the advantages of a standardized testing technique and data analysis procedure would be realized, but the performance parameters would be far more meaningful and capable of providing much greater insight into the underlying factors which actually determine aerator efficiency. Translation of the unsteady-state performance results to actual process design operating conditions would also be much improved and more straightforward.

ASCE Standard Model for Oxygen Mass Transfer in Aeration Systems

The ASCE Standard for the measurement of aerator oxygen transfer efficiency in clean tap water is based on the unsteady-state reaeration technique. Using this approach, the body of clean test water is first deoxygenated using the cobalt-catalyzed sodium sulfite reaction and is then reaerated back to "saturated" or "steady-state" conditions with accurate experimental measurement of the increase of the test water dissolved oxygen (DO) level with time. The ASCE Standard has made great strides in precisely defining the exact experimental procedures and techniques to be followed in gathering the unsteady-state reaeration test data.

The oxygen mass transfer model used in the ASCE Standard and commonly accepted for many years in the unsteady-state testing of aerators is as follows:

$$\frac{dC_L}{dt} = k_L a(C^*_\infty - C_L) \tag{1}$$

This is a liquid-phase model which ignores conditions in the gas phase and is used in identical form for diffused or subsurface aeration systems and surface aerators alike. If $k_L a$ and C^*_∞ are assumed constant throughout the unsteady-state testing period, Eq. 1 can be readily integrated to yield the following expression for C_L as a function of time.

$$C_L = C^*_\infty - (C^*_\infty - C_o)\exp(-k_L a\, t) \tag{2}$$

Using the ASCE Standard, a nonlinear regression is performed on the test data to give the "best statistical fit" of the experimental DO concentration vs. time data to Eq. 2. In this way, values of $k_L a$, C^*_∞, and C_o are obtained for each experimental DO level data set for each specific test condition. The values of $K_L a$ and C^*_∞ are empirically adjusted to 20°C and then the average standard oxygen transfer rate (SOTR) and standard aeration efficiency (SAE) can be calculated. The SOTR is the mass of oxygen transferred per unit of time at a bulk liquid DO level of 0 mg/L, 20°C, and 1-atm pressure. The SAE is the mass of oxygen transferred per unit of time at standard conditions per unit of total power input. The more common units employed for SOTR and SAE in the industry are pounds of oxygen transferred per hour and pounds of oxygen per hour per horsepower, respectively. The power input for a diffused or subsurface aeration system includes blower power as well as shaft power input if a mechanical aerator is used.

Assumptions and Limitations of the ASCE Standard Oxygen Mass Transfer Model

The use of Eqs. 1 and 2 to characterize the oxygen mass transfer process for diffused air and subsurface mechanical aeration systems embodies a large number of simplifying assumptions which require critical examination relative to the actual mass transfer process involved. The primary assumptions made in the use of Eqs. 1 and 2 involve the following:

1. The bulk test liquid under aeration is completely mixed (of a uniform DO concentration throughout the tank) at any instant of time during the test period.
2. The oxygen mass transfer process is predominantly liquid-phase mass transfer controlled, and the gas-phase resistance to mass transfer can be ignored.
3. The mass transfer of all gaseous components other than oxygen (including nitrogen and water vapor) has a negligible impact on the oxygen mass transfer process and can be ignored.
4. A single mass transfer coefficient and driving force term can adequately describe the overall oxygen transfer process.
5. A single and constant value of C^*_∞, the "saturation" or "steady-state" DO concentration, in the driving force term is an adequate representation of the equilibrium DO concentration for oxygen transfer into the liquid phase and applies to the entire aeration test tank throughout the unsteady-state reaeration test period.

Assumptions 1 to 3 are generally quite good and have been shown to apply reasonably well under most aeration system test conditions. The complete mixing assumption is probably not too closely followed at the start of the reaeration test when the bulk liquid DO level is near zero and the oxygen transfer rate is very high, but extensive multiple-point DO sampling in aeration test systems has shown that this assumption is closely followed throughout most of the unsteady-state reaeration test period for properly-configured test tank systems. The gas-liquid mass transfer of all sparingly soluble gases into water is predominantly liquid-phase mass transfer controlled and oxygen is certainly no exception. The simultaneous mass transfer of nitrogen from the aeration gas into the test water and *vice versa* certainly occurs in deep tank aeration systems, but its impact on the oxygen transfer process can be shown to be negligible under most conditions.

The primary limiting constraints involved in the use of Eqs. 1 and 2 to analyze the oxygen mass transfer process in diffused air and subsurface mechanical aeration systems are related to assumptions 4 and 5. As will be shown, a single mass transfer coefficient and driving force term cannot accurately describe the overall oxygen transfer process and the equilibrium DO level for mass transfer in these systems is also a strong function of the liquid depth (pressure level) and the oxygen concentration of the aeration gas. Consequently, the use of a single and constant "saturation" or "steady-state" value of C^*_∞ is not a meaningful equilibrium DO concentration of any kind, but rather nothing more than a statistically-derived factor from the nonlinear regression analysis. Thus, both the values of $k_L a$ and C^*_∞ are simply statistically derived correlation factors and are not true or fundamentally-defined mass transfer coefficients or equilibrium DO concentrations. In this context, the values of $k_L a$ and C^*_∞ so determined have meaning only as a pair of inextricably-derived correlation factors which cannot be separately analyzed or evaluated. They do not represent a meaningful oxygen mass transfer relationship for diffused air or subsurface mechanical aeration systems.

Development of Improved Oxygen Mass Transfer Model

The development of an effective mass transfer model for the subject types of aeration systems must first start with a realistic physical description of the actual gas-liquid contacting system involved. Figure 1 presents a front cross-sectional view of a diffused air aeration tank. The aeration gas is diffused into the liquid near the bottom of the tank and flows upward through the liquid to the surface of the tank. The bubbling action of the gas produces effective bulk motion and mixing of the liquid in the tank and also produces a rather turbulent liquid surface. Two separate mass transfer zones and mechanisms of oxygen transfer exist as identified in Figure 1. These are the gas-bubble dispersion mass transfer zone which exists below the liquid surface in the tank and the turbulent liquid surface reaeration mass transfer zone at the surface of the tank liquid. Each of these mass transfer zones must be separately analyzed and properly accounted for in the overall oxygen mass transfer model.

Gas bubble dispersion mass transfer zone

This zone can be intuitively recognized as probably accounting for most of the oxygen mass transfer in diffused-air and subsurface mechanical aeration systems and is characterized by gas bubbles dispersed throughout the continuous bulk liquid phase. Agitation and mixing of the bulk liquid phase by either the action of the gas bubbles and/or a mechanical mixing impeller is quite good, and the bulk liquid in the tank can be reasonably well assumed to be perfectly mixed and of a uniform composition throughout at any instant of time. The gas bubbles can also be assumed to flow up through the liquid in essentially a plug flow fashion. Certainly there is some minimal degree of mixing and recirculation of gas bubbles within the bulk liquid phase, but the overall impact of this effect is felt to be negligible. The essence of the gas-bubble dispersion mass transfer zone is that of a completely-mixed liquid phase with uniform plug flow of gas bubbles up through the liquid. The only other assumptions, which will be made for this mass transfer zone, are that the oxygen transfer process is liquid-phase-controlled and the transfer of gaseous components other than oxygen can be ignored. It will, of course, also be assumed that the gas feed rate into the system is constant and that the overall hydrodynamic conditions within the aeration tank are stable and invariant with time. These later assumptions are also inherently part of the simplified ASCE model.

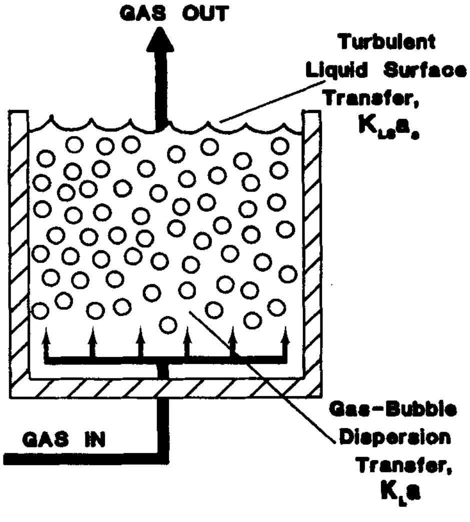

Figure 1. Diffused-air or subsurface aeration system mass transfer zones.

With the above assumptions, the gas-bubble dispersion mass transfer zone model can be readily derived by consideration of a fixed position, differential vertical contacting section within the two-phase system. Within this differential contacting element the liquid is of a uniform DO level, C_L, and the gas-phase bubbles traverse through this section in plug flow fashion. A gas-phase oxygen material balance for this differential contacting section then yields the following relationship.

$$K_1 \frac{\partial(H_g y)}{\partial t} = -\frac{\partial(Gy)}{A_T \partial Z} - k_L a(C_L^* - C_L)K_2 \qquad (3)$$

Since G and H_g can be reasonably assumed to be constant and independent of a position within the tank, Eq. 3 can be simplified as follows:

$$K_1 H_g \frac{\partial y}{\partial t} = -\frac{G}{A_T}\frac{\partial y}{\partial Z} - k_L a(C_L^* - C_L)K_2 \qquad (4)$$

Equation 4 thus describes the gas-phase oxygen composition, y, as a function of time and depth within the aeration tank. The mass transfer coefficient $k_L a$ is a correctly- and fundamentally-defined mass transfer coefficient as is the value of C_L^*, the true bulk liquid equilibrium DO level at any position, Z, and gas-phase composition, y, in the tank. C_L^* obeys Henry's law and as such is a function of the gas-phase oxygen concentration, y, and the depth Z (pressure level) at any given water temperature. The value of C_L^* can be calculated at any depth Z and gas-phase composition, y, within the tank by the following expression.

$$C_L^* = C_{ST}^* \left(\frac{P_B - P_V + \dfrac{(Z_S - Z)}{10.33}}{1.0 - P_V} \right) \frac{y}{0.266} \qquad (5)$$

The boundary condition on Eq. 4 is that $y = 0.266$ at $Z = 0$ for all values of t. The initial condition ($t = 0$) values of y as a function of Z are determined from the following equation.

$$0 = -\frac{G}{A_T}\frac{dy}{dZ} - k_L a\, C_L^* K_2 \qquad (6)$$

This steady-state equation describes the y profile (y vs. Z) in the tank at zero bulk liquid DO which approximates the actual initial conditions after deoxygenation of the tank liquid with excess sodium sulfite during the reaeration test. This initial-condition y profile was used for all calculations using Eq. 4.

Equations 4 and 5 must now be combined with an overall oxygen material balance equation for the bulk liquid phase which will give the value of C_L as a function of time. Before this is done, however, we must also consider the oxygen transfer from the surface reaeration mass transfer zone to the bulk liquid within the tank.

Surface reaeration mass transfer zone

This mass transfer zone involves the transfer of oxygen from atmospheric air to the turbulent liquid surface of the aeration tank. The turbulent liquid surface is created, of course, by the vigorous bubbling action of the air within the tank and/or the surface turbulence created by the action of a submerged mixing impeller. The transfer rate for this mechanism of oxygen dissolution can be simply described by the following additional mass transfer rate expression.

$$STR = k_{LS}A_S(C^*_{LS} - C_L) \tag{7}$$

where STR is the surface reaeration oxygen mass transfer rate in mg/L/h. The value of C^*_{LS} is determined from equilibrium with atmospheric air as opposed to the exit gas composition from the aeration tank because of the low superficial gas velocity in aeration tanks. As a consequence, the exit gas will be readily dispersed into the atmospheric air above the tank, and the turbulent liquid surface will essentially be in contact with gas of atmospheric air composition.

As with the use of k_La in Eq. 4 for the gas bubble dispersion mass transfer zone, the surface reaeration mass transfer coefficient, k_{LS}, is a fundamentally-defined overall liquid-phase mass transfer coefficient as is the value of C^*_{LS}, the equilibrium DO level at the liquid surface. C^*_{LS} can be calculated from Eq. 5 with $Z = Z_S$ and $y = 0.266$.

Oxygen uptake rate of bulk liquid phase

The overall oxygen uptake rate of the bulk liquid phase within the test tank can now be determined by adding the transfer rate from the gas-bubble dispersion and the surface reaeration mass transfer zones. An overall oxygen mass balance for the bulk liquid phase in the tank then gives the following expression.

$$V_L\frac{dC_L}{dt} = \int_0^{Z_S} k_La(C^*_L - C_L)\,A_T\,dZ + k_{LS}A_S(C^*_{LS} - C_L) \tag{8}$$

Rearranging and simplifying,

$$\frac{dC_L}{dt} = \int_0^{Z_S} \frac{k_La(C^*_L - C_L)\,dZ}{Z_S} = k_{LS}a_S(C^*_{LS} - C_L) \tag{9}$$

Where

$$k_{LS}a_S = \frac{k_{LS}A_S}{V_L},\ \mathrm{h}^{-1} \tag{10}$$

Equation 9 then gives C_L as a function of time throughout the unsteady-state reaeration test period. The initial condition in Eq. 9 is $C_L = C_0$ at $t = 0$.

Equations 4, 5 and 9 now completely define the unsteady-state reaeration mass transfer model for diffused air or subsurface mechanical aeration systems. These equations contain only two unknown parameters, k_La and $k_{LS}a_S$ which can be correlated with the experimental C_L vs. time data to obtain the "best-fit" values of k_La and $k_{LS}a_S$. The numerical integration of Eqs. 4 and 9 is quite straightforward using standard numerical techniques (Chapra and Canale, 1988; Smith, 1985).

Equation 4, however, can be further simplified without sacrificing much accuracy in the analysis by recognizing that the value of H_g is usually quite low and, consequently, the value of the partial derivative term with respect to time in Eq. 12 can be readily ignored. With this simplification, Eq. 4 becomes

$$\frac{dy}{dZ} = -\left[\frac{A_T}{G}\right] K_La\,(C^*_L - C_L)\,K_2 \tag{11}$$

Calculated results comparing the use of Eqs. 4 and 11 show virtually the same results in all cases.

We now have two coupled, nonlinear ordinary differential equations (Eqs. 9 and 11) which enable a much simpler numerical integration procedure to be used. The values of k_La and $k_{LS}a_S$ are, of course, assumed to be constant during the integration of Eqs. 9 and 11. The values of k_La and $k_{LS}a_S$ should be only a function of the hydrodynamic conditions within the test tank and, therefore, invariant with time for a given set of diffused aeration system air flow rates and submerged mechanical aeration system operating conditions.

Results: New Model vs. ASCE Standard Model

The improved model was compared with the ASCE Standard Model using actual experimental diffused-air reaeration test data obtained from the County Sanitation Districts of Los Angeles County under EPA Contract 14-12-150 (Yunt and Hancuff, 1979). These data are considered to be some of the more meticulously obtained and accurate diffused-air reaeration test data ever gathered and consequently were of great value in testing the validity of the new oxygen mass transfer model. All of the experimental dissolved oxygen data in this EPA test program were obtained using liquid samples pumped from the aeration test basin and Winkler titrations to determine the DO levels (Standard methods). The data are therefore felt to be highly reliable. Table 1 summarizes the testing conditions

Table 1. Testing Conditions and Equipment Parameters for Diffused Air Reaeration

Aeration Testing Conditions

Barometric Pres.: 0.9942–1.006 atm
Water Temp.: 16.2–23.2°C
Rel. Humidity: 29.9–82.8%
Ambient Air Temp.: 16.6–28.9°C
Tank Surface Area: 37.2 m²
Tank Liquid Depth: 3.05–7.62 m
Tank Liquid Vol.: 113,200–283,100 L

Coarse Bubble Diffuser System

Each diffuser is 0.16 m above the tank floor. At nominal depths of 4.57 and 7.62 m, 30 equally-spaced diffusers were located off a central header along the center line of the tank floor. At nominal depths of 3.05 and 6.10 m, a total of 24 diffusers, eight on each header 1.02 m from the wall and eight on a central header along the center line of the tank floor. Each diffuser was equally spaced on all the headers; air flow range, 3.55–11.39 m³/min.

Fine Bubble Diffuser System

126 diffusers in full floor coverage, uniformly spaced 0.28 m above tank floor; air flow range, 2.09–7.71 m³/min.

Table 2. Improved Model vs. ASCE Standard Model for Fine Bubble Diffusers

Sample Point	k_La	$k_{LS}a_S$	Residual Sum of Sq. SSQ
	New Mass Transfer Model		
A	8.1	1.6	0.029
B	7.8	1.8	0.043
C	9.3	0.8	0.015
D	8.8	1.1	0.205
Tank Avg.	8.7	1.1	0.034

Sample Point	k_La	C^*_∞	Residual Sum of Sq. SSQ
	ASCE Standard Model		
A	8.3	11.7	0.025
B	8.2	11.6	0.044
C	8.4	11.9	0.026
D	8.3	12.0	0.251

Air flow rate: 3.61 m^3/min
Nominal tank liquid depth: 7.62 m

and equipment parameters for the experimental reaeration test data. As shown, representative data for both fine bubble and coarse bubble diffused-air systems were analyzed using the new model over a wide range of tank liquid depths and air flow rates.

Tables 2 and 3 summarize a typical comparison of the experimental data correlations using the new model and the ASCE Standard Model for both the fine bubble and coarse bubble diffuser tests. As shown, the new model correlates the experimental data as well as or better than the ASCE Standard Model from a statistical standpoint even though the new model is far more rigorous and restrictive in nature. The ASCE model is a three-parameter model (k_La, C^*_∞ and C_o), whereas the new model regresses on only two parameters, k_La and $k_{LS}a_S$. The values of C^*_L and C^*_{LS} are defined totally by Henry's law (Eq. 5) in the new model, whereas the value of C^*_∞ is adjusted arbitrarily in the ASCE Standard Model to achieve the best possible fit of the data. As a consequence, of course, the values of k_La and C^*_∞ are not independent parameters in the ASCE Model as pointed out previously. The values of k_La and C^*_∞ are simply empirical constants determined in an exponential relationship whose values are completely free to provide the best possible statistical fit of the experimental data.

Table 3. Improved Model vs. ASCE Standard Model for Coarse Bubble Diffusers

Sample Point	k_La	$k_{LS}a_S$	Residual Sum of Sq. SSQ
	New Mass Transfer Model		
A	2.3	0.9	0.047
B	2.3	0.9	0.035
C	2.3	0.9	0.059
D	2.3	0.9	0.020
Tank Avg.	2.3	0.9	0.020

Sample Point	k_La	C^*_∞	Residual Sum of Sq. SSQ
	ASCE Standard Model		
A	3.4	11.5	0.035
B	3.3	11.5	0.028
C	3.3	11.6	0.116
D	3.2	11.6	0.061

Air flow rate: 3.64 m^3/min
Nominal tank liquid depth: 7.62 m

Tables 2 and 3 also show that there is good agreement among all the sample point locations with the new model as determined by independent analysis of the individual sample point data. Comparison of the results from the individual sample point locations with the total tank average data is particularly good for the new model.

Figure 2 shows a typical plot of the experimental C_L vs. time test data compared to the calculated values of C_L vs. time using the results of the new mass transfer model. The plotted points are the experimental data, and the solid curve gives the calculated values of C_L vs. time. As would be expected from the very good statistical fit of the data, the calculated and experimental data agree to well within the probable experimental error of the dissolved oxygen data which would be estimated at about ±0.05 mg/L.

Another advantage of the new model in comparison to the current ASCE Standard in this regard is with respect to the experimental test data requirements. The ASCE Standard Model requires rather extensive and accurate data at DO levels closely approaching the "saturation" or "steady-state" DO level. These data are necessary in order to obtain a good representative value of C^*_∞, which is, of course, absolutely critical to the accuracy of the results using the ASCE Model. This need is not nearly as great with the improved mass transfer model, since the equilibrium DO levels are calculated exactly at all times throughout the reaeration process, and the final "steady-state" DO level is calculated directly by the model.

The steady-state or so-called "saturation" DO level ultimately reached in deep tank diffused-air reaeration tests has also been a source of much confusion over the years. The ASCE Model gives virtually no insight into this matter; however, the new and improved mass transfer model completely defines and clarifies this situation. Figure 3 shows a plot of gas-phase oxygen compositions (y vs. Z profiles) as a function of time during a

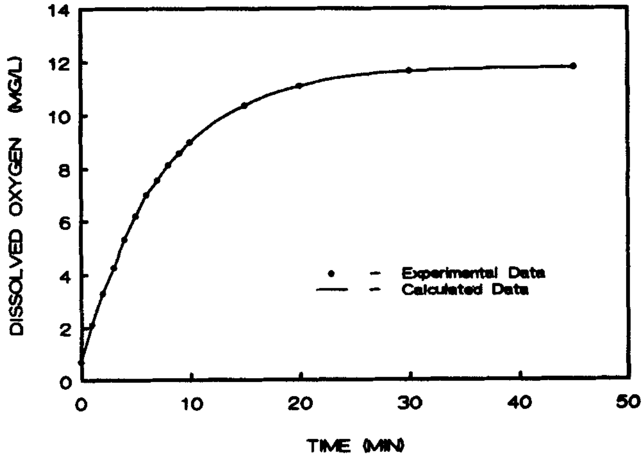

Figure 2. Unsteady-state reaeration test results using new mass transfer model: experimental vs. calculated.

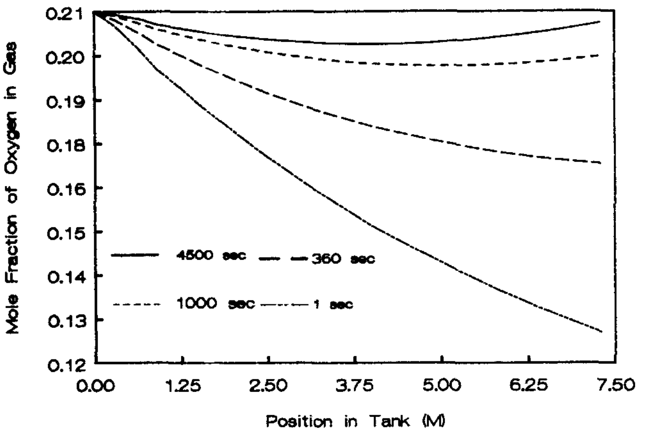

Figure 3. Gas-phase composition calculated for unsteady-state fine-bubble reaeration test using new mass transfer model.

typical deep tank fine-bubble diffused-air reaeration test. At the start of the test at low bulk liquid DO levels, the overall oxygen uptake rate is rather high and the percentage oxygen absorption from the gas phase is quite appreciable. Oxygen absorption efficiencies between 40 and 50% are realized with some fine bubble diffuser systems at bulk liquid DO levels near zero. This condition is shown in Figure 3 for the gas-phase oxygen composition profile at a time of one second. As reaeration continues, however, the bulk liquid DO level rapidly increases and the percentage oxygen absorption decreases accordingly as depicted in Figure 3.

A steady-state DO level is ultimately reached when the oxygen gas-phase composition profile becomes as shown for the curve at 4,500 seconds in Figure 3. At the steady-state bulk liquid DO level, oxygen is transferred from the gas phase to the liquid phase at the deeper depths in the tank and, conversely, oxygen is desorbed from the liquid into the gas phase at the shallower depths in the tank. Oxygen is also simultaneously being desorbed from the liquid surface into the atmosphere in the surface reaeration mass transfer zone at the steady-state DO level. The steady-state bulk liquid DO level then corresponds to the point at which the absorption of oxygen from the aeration gas into the liquid from the deeper tank depths is exactly balanced by the desorption of oxygen from the liquid into the gas in the shallower tank depths and the surface reaeration zone desorption of oxygen into the atmospheric air. The desorption of oxygen from the liquid surface of the tank into the atmospheric air at the steady-state point, of course, always causes the steady-state DO level reached to be equivalent to an equilibrium saturation depth of less than the midtank depth. This phenomenon has been observed for years by many investigators, but never adequately understood or explained. This result becomes intuitively obvious with the inclusion of the appropriate surface reaeration term in the overall mass transfer model.

The infinite time or ultimate steady-state DO level reached in a deep-tank diffused-air reaeration test is, therefore, not an equilibrium or "saturation" DO level at all. It is the dynamic steady-state bulk liquid DO level at which the absorption of oxygen from the gas phase in the deep portions of the tank is exactly offset by the desorption of oxygen to the gas phase in the upper portions of the tank plus the desorption of oxygen from the surface of the tank liquid into the atmospheric air. This dynamic steady-state mass transfer condition is totally described by setting the value of the derivative of C_L with respect to time in Eq. 9 equal to zero.

$$\frac{dC_L}{dt} = 0 = \int_0^{Z_S} \frac{k_La(C_L^* - C_L)dZ}{Z_S} + k_{LS}a_S(C_{LS}^* - C_L) \quad (12)$$

Rearranging,

$$k_{LS}a_S(C_L - C_{LS}^*) = \frac{k_La}{Z_S}\int_0^{Z_S} (C_L^* - C_L)dZ \quad (13)$$

At the dynamic steady-state DO level in a deep tank diffused-air aeration system, C_L is greater than C_{LS}^* and, therefore, the lefthand side of Eq. 13 is positive as is the value of the integral on the righthand side of the equation. This is why the steady-state value of C_L corresponds to an equivalent equilibrium saturation depth of less than the tank's midpoint depth. The value of the integral on the righthand side of Eq. 13 would be equal to zero at a value of C_L corresponding to the middepth equilibrium saturation point.

Gas-Bubble Dispersion vs. Surface Reaeration Zone Mass Transfer Coefficients

Comparison of the gas-bubble dispersion zone mass transfer coefficients with the surface reaeration zone mass transfer coefficients for the fine bubble and coarse bubble diffuser systems is also quite revealing. As shown in Tables 2 and 3, the gas-bubble dispersion volumetric mass transfer coefficient for the fine bubble diffusion system is more than three times higher than that for the coarse bubble system when operating at essentially the same gas flow rate and tank depth. This is undoubtedly due to the smaller bubble size and higher interfacial area per unit volume of gas flow produced by the fine bubble diffuser system. The surface reaeration zone mass transfer coefficients, on the other hand, are about the same order of magnitude for both types of diffused aeration system. This would indicate that the degree of surface turbulence and liquid surface interfacial area for the two systems are about the same at comparable gas flow rates and liquid depths. This result is not too surprising when one considers the comparable degree of surface liquid turbulence and motion produced by the two types of diffuser systems at the same gas flow rates.

The magnitude of the aeration tank surface reaeration zone mass transfer coefficients determined by the new model can also be readily compared with surface reaeration oxygen mass transfer coefficients measured for shallow streams with liquid velocities in the range of several feet per second. It is estimated that the surface liquid velocities in diffused aeration tanks are probably of the same order of magnitude although the velocity profiles would certainly be more complex than those encountered in shallow streams. Nevertheless, it is interesting to note that the surface reaeration oxygen mass transfer coefficients for 1-ft-deep (0.3-m) streams with 1- to 4-ft/s (3- to 12-m/s) water velocities have been measured in the range of 0.9 to 2.3 h^{-1} (Churchill et al., 1962; Owens et al., 1964; Thomann and Mueller, 1987). This is certainly comparable to the value of the surface reaeration zone oxygen mass transfer coefficients determined from diffused-air aeration tanks using the new model.

Figure 4 summarizes the new mass transfer model correlations for both the fine bubble diffuser and coarse bubble diffuser

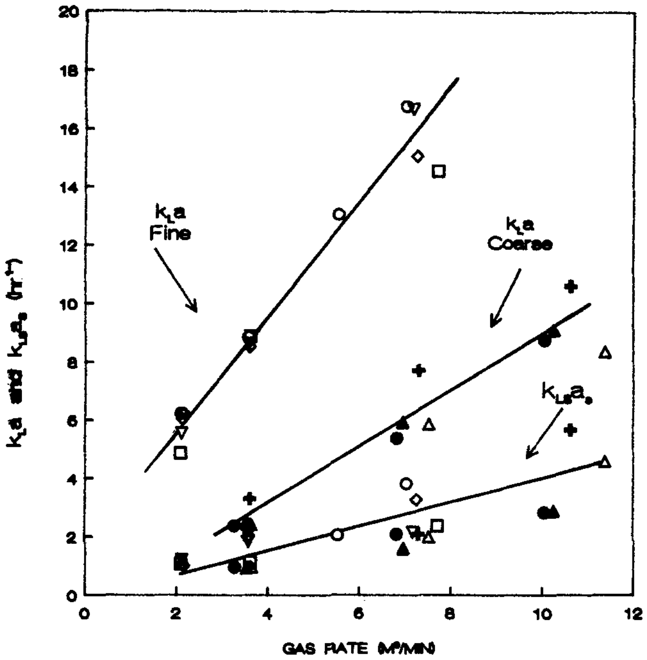

Figure 4. Mass transfer coefficients using new mass transfer model: fine bubble vs. coarse bubble.

△ 7.62 M Coarse + 6.10 M Coarse ▲ 4.57 M Coarse ● 3.05 M Coarse

□ 7.62 M Fine ◇ 6.10 M Fine ▽ 4.57 M Fine ○ 3.05 M Fine

test results analyzed. As shown, the values of both the gas-bubble dispersion mass transfer coefficients and the surface reaeration zone mass transfer coefficients can be linearly correlated reasonably well with gas flow rate for both the fine bubble and coarse bubble diffuser systems for all tank liquid depths between 3.05 and 7.62 m. This means that the volumetric oxygen mass transfer coefficients vary in direct proportion to the gas flow rate for both types of diffusers and are essentially independent of liquid depth. This result is intuitively appealing, since the values of k_La and $k_{LS}a_S$ would be expected to be influenced only by the hydrodynamic conditions in the tank which are predominantly determined by the gas flow rate and not the tank liquid depth. The ratio of k_La to $k_{LS}a_S$ for fine bubble diffused-air systems is also much larger than that for coarse bubble diffused-air systems as should be anticipated. This ratio varies from about 5 to 8 for the fine bubble diffuser system analyzed vs. a range of from 2 to 3 for the coarse bubble diffuser systems.

The gas-bubble dispersion mass transfer coefficients for the fine bubble diffuser system are about five times larger than those for the coarse bubble diffuser system at low gas flow rates and about three times larger at the high gas flow rates. The surface reaeration mass transfer coefficients, however, are of comparable values for both systems at the same gas flow rates. This result is also intuitively appealing, since this mass transfer mechanism should be expected to be reasonably comparable for both systems at the same gas flow rates and levels of bulk liquid surface motion and turbulence.

Standard Oxygen Transfer Rate under Process Design Operating Conditions Using the New Mass Transfer Model

Translation of the unsteady-state reaeration test results to the standard oxygen transfer rate (SOTR) and the oxygen transfer rate under actual steady-state process design operating conditions (OTR) is also considerably simplified using the new and improved mass transfer model. The SOTR is directly calculated from Eqs. 9 and 11 by substituting the value of the mass transfer coefficients at 20°C, setting $C_L = 0$ and multiplying each mass transfer term by V_L.

$$\mathrm{SOTR} = V_L \int_0^{Z_s} \frac{k_L a_{20} C_L^*}{Z_s}\, dz + V_L k_{LS} a_{s,20} C_{LS}^* \tag{14}$$

$$\frac{dy}{dz} = -\left(\frac{A_t}{G}\right) k_L a_{20} C_L^* K_2 \tag{15}$$

C_L^* and C_{LS}^*, as always with the new model, are calculated from Eq. 5. Equations 14 and 15 are then directly integrated numerically to yield the SOTR value. The approximate equations used in the ASCE Standard to correct from the actual aeration test conditions to standard conditions are totally avoided. These parameters are accounted directly for in Eqs. 14 and 15. A temperature correction factor must still be used, however, to correct k_La_T to k_La_{20} and $K_{LS}a_{s,T}$ to $k_{LS}a_{s,20}$. The development of a more fundamentally-rigorous basis for correcting the value of k_La_T and $K_{LS}a_{s,T}$ for the effect of liquid temperature is still needed.

The oxygen transfer rate under actual steady-state process design operating conditions is determined from Eqs. 9 and 11 by setting C_L equal to the desired bulk liquid DO level, C_S, and by employing the usual definitions of α and β to convert from clean tap water conditions to the actual process design wastewater conditions.

$$\mathrm{OTR} = V_L \int_0^{Z_s} \frac{\alpha_1 k_L a_T (\beta C_L^* - C_S)}{Z_S}\, dZ + V_L \alpha_2 k_{LS} a_{s,T} (\beta C_{LS}^* - C_S) \tag{16}$$

$$\frac{dy}{dZ} = -\left(\frac{A_t}{G}\right) \alpha_1 k_L a_T (\beta C_L^* - C_s) K_2 \tag{17}$$

$$\alpha_1 = \frac{K_L a_T \text{ (process water)}}{K_L a_T \text{ (clean tap water)}} \tag{18}$$

$$\alpha_2 = \frac{k_{LS} a_{S,T} \text{ (process water)}}{k_{LS} a_{S,T} \text{ (clean tap water)}} \tag{19}$$

$$\beta = \frac{C_L^* \text{ (process water)}}{C_L^* \text{ (clean tap water)}} = \frac{C_{LS}^* \text{ (process water)}}{C_{LS}^* \text{ (clean tap water)}} \tag{20}$$

As shown in Eqs. 18 and 19, however, the new model incorporates separate α factors for each of the individual mass transfer coefficients for the two separate mass transfer zones. The values of these separate α factors would be determined in the usual fashion from unsteady reaeration measurements conducted in the actual process wastewater, and comparison of these measurements with clean tap water results. The values of α_1 and α_2 could be quite different because of the fundamentally different levels of interfacial turbulence and mechanism of gas-liquid interphase mass transfer between the two mass transfer zones.

Conclusions

A new oxygen mass transfer model, developed for diffused-air and subsurface mechanical aeration systems, represents a significant improvement over current empirical models widely used in the industry. The new model fundamentally offers a more rigorous description of the actual mass transfer process which occurs in these systems and includes an important surface reaeration zone mass transfer term which has not been included in all previously-used models. The new two-parameter model correlates experimental unsteady-state reaeration test data in clean tap water at least as well as, if not better than, the currently-used empirical, three-parameter ASCE Standard Model and provides much greater insight into the actual process dynamic variations occurring during unsteady-state reaeration tests. The new model also is more accurate and easy to use in translating clean tap water aeration system test results into performance under actual steady-state process design operating conditions.

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Notation

A_S = turbulent liquid surface interfacial area for oxygen mass transfer to bulk liquid from surface reaeration mass transfer zone, m^2
A_T = tank cross-sectional area, m^2
C_L = bulk liquid DO level at any time t, mg/L
C_L^* = true bulk liquid equilibrium DO level at any liquid temperature T, barometric pressure P_B, gas-phase oxygen composition y, and depth Z in test tank, mg/L
C_∞^* = statistically-defined "saturation" DO level for bulk liquid at infinite time in ASCE Model, mg/L
C_o = initial bulk test liquid DO level, mg/L
C_{ST}^* = standard methods DO level in equilibrium with atmospheric air at test liquid temperature, standard atmospheric pressure, 100% relative humidity, mg/L
C_{LS}^* = true equilibrium DO level at turbulent liquid surface at liquid temperature T and barometric pressure P_B in equilibrium with atmospheric air, mg/L
C_S = steady-state bulk liquid DO level under actual process design conditions, mg/L
d_e = effective saturation depth, m
G = gas flow rate, kmol·N_2/h
H_g = dispersed gas-phase holdup per unit volume of dispersion, m^3N_2/m^3
K_1 = unit conversion factor, kmol N_2/m^3N_2
K_2 = unit conversion factor, 3.13×10^{-5} kmol $O_2 \cdot L/m^3 \cdot mg$
k_La = overall volumetric liquid-phase mass transfer coefficient for gas bubble dispersion mass transfer zone, h^{-1}; statistically-defined "apparent " mass transfer coefficient in ASCE model, h^{-1}
k_{LS} = surface reaeration zone liquid-phase mass transfer coefficient, m/h
$k_{LS}a_S$ = overall volumetric surface reaeration oxygen mass transfer coefficient, h^{-1}
n = number of DO concentration vs. time data sets for each test condition
OTR = oxygen transfer rate under actual steady-state process design conditions, mg/h
P_B = barometric pressure, atm
P_S = standard conditions barometric pressure of one atmosphere, atm
P_V = vapor pressure of water at test liquid temperature, atm
SAE = standard aeration efficiency, mg 0_2/W·h
SOTR = standard oxygen transfer rate, mg 0_2/h
STR = surface reaeration oxygen mass transfer rate, mg 0_2/h
T = liquid temperature, °C
t = time, h
V_L = liquid volume in aeration tank, L or m^3
y = oxygen concentration in gas phase, kmol 0_2/kmol N_2
Z = vertical depth position in tank, $Z = 0$ at gas inlet, $Z = Z_s$ at liquid surface, m

Greek letters

α_1 = ratio of K_La_T in actual process wastewater to K_La_T in clean tap water
α_2 = ratio of $K_{LS}a_{S,T}$ in actual process wastewater to $K_{LS}a_{S,T}$ in clean tap water
β = ratio of equilibrium DO levels in actual process wasterwater to equilibrium DO levels in clean tap water

Subscripts

T = temperature of liquid, °C
20 = standard temperature, 20°C
L = liquid phase
∞ = infinite time value
ST = standard methods value
0 = time zero value
LS = liquid surface
S = surface value or standard value
g = gas phase
v = water vapor

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